15 Mechanical Properties of Biological Tissues

15.1 Viscoelasticity

The material response discussed in the previous chapters was limited to the response of elastic materials, in particular to linearly elastic materials. Most metals, for example, exhibit linearly elastic behavior when they are subjected to relatively low stresses at room temperature. They undergo plastic deformations at high stress levels. For an elastic material, the relationship between stress and strain can be expressed in the following general form:

$$\sigma = \sigma(\varepsilon). \tag{15.1}$$

Equation (15.1) states that the normal stress σ is a function of normal strain ε only. The relationship between the shear stress τ and shear strain γ can be expressed in a similar manner. For a linearly elastic material, stress is linearly proportional to strain, and in the case of normal stress and strain, the constant of proportionality is the elastic modulus E of the material (Fig. 15.1):

$$\sigma = E\varepsilon. \tag{15.2}$$

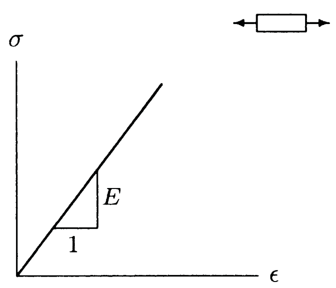

Fig. 15.1 Linearly elastic material behavior

While investigating the response of an elastic material, the concept of time does not enter into the discussions. Elastic materials show time-independent material behavior. Elastic materials deform instantaneously when they are subjected to externally applied loads. They resume their original (unstressed) shapes almost instantly when the applied loads are removed.

There is a different group of materials—such as polymer plastics, almost all biological materials, and metals at high temperatures—that exhibits gradual deformation and recovery when they are subjected to loading and unloading. The response of such materials is dependent upon how quickly the load is applied or removed, the extent of deformation being dependent upon the rate at which the deformation-causing loads are applied. This time-dependent material behavior is called *viscoelasticity*. Viscoelasticity is made up of two words: viscosity and elasticity. *Viscosity* is a fluid property and is a measure of resistance to flow. *Elasticity*, on the other hand, is a solid material property. Therefore, a viscoelastic material is one that possesses both fluid and solid properties.

For viscoelastic materials, the relationship between stress and strain can be expressed as:

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}). \tag{15.3}$$

Equation (15.3) states that stress, σ, is not only a function of strain, ε, but is also a function of the *strain rate*, $\dot{\varepsilon} = \mathrm{d}\varepsilon/\mathrm{d}t$, where t is time. A more general form of Eq. (15.3) can be obtained by including higher order time derivatives of strain. Equation (15.3) indicates that the stress–strain diagram of a viscoelastic material is not unique but is dependent upon the rate at which the strain is developed in the material (Fig. 15.2).

N. Özkaya et al., *Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation*,
DOI 10.1007/978-1-4614-1150-5_15,

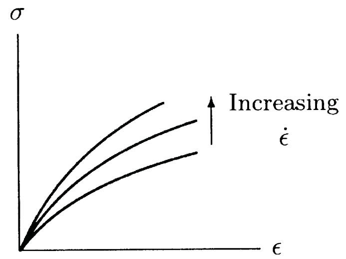

Fig. 15.2 Strain rate ($\dot{\varepsilon}$) dependent viscoelastic behavior

15.2 Analogies Based on Springs and Dashpots

In Sect. 13.8, while covering Hooke's Law, an analogy was made between linearly elastic materials and linear springs. An elastic material deforms, stores potential energy, and recovers deformations in a manner similar to that of a spring. The elastic modulus E for a linearly elastic material relates stresses and strains, whereas the constant k for a linear spring relates applied forces and corresponding deformations (Fig. 15.3). Both E and k are measures of stiffness. The similarities between elastic materials and springs suggest that springs can be used to represent elastic material behavior. Since these similarities were first noted by Robert Hooke, elastic materials are also known as *Hookean solids.*

When subjected to external loads, fluids deform as well. Fluids deform continuously, or *flow*. For fluids, stresses are not dependent upon the strains but on the strain rates. If the stresses and strain rates in a fluid are linearly proportional, then the fluid is called a *linearly viscous fluid* or a *Newtonian fluid*. Examples of linearly viscous fluids include water and blood plasma. For a linearly viscous fluid,

$$\sigma = \eta(\dot{\varepsilon}). \tag{15.4}$$

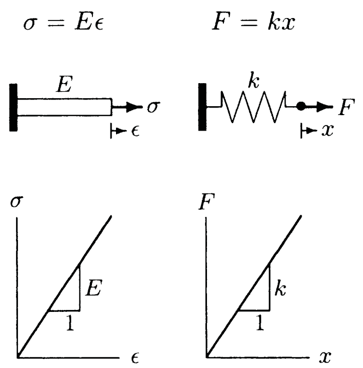

Fig. 15.3 Analogy between a linear spring and an elastic solid

In Eq. (15.4), η (eta) is the constant of proportionality between the stress σ and the strain rate $\dot{\varepsilon}$, and is called the *coefficient of viscosity* of the fluid. As illustrated in Fig. 15.4, the coefficient of viscosity is the slope of the $\sigma - \dot{\varepsilon}$ graph of a Newtonian fluid. The physical significance of this coefficient is similar to that of the coefficient of friction between the contact surfaces of solid bodies. The higher the coefficient of viscosity, the "thicker" the fluid and the more difficult it is to deform. The coefficient of viscosity for water is about 1 centipoise at room temperature, while it is about 1.2 centipoise for blood plasma.

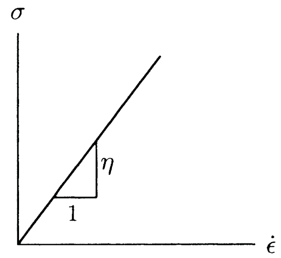

Fig. 15.4 Stress–strain rate diagram for a linearly viscous fluid

The spring is one of the two basic mechanical elements used to simulate the mechanical behavior of materials. The second basic mechanical element is called the *dashpot*, which is used to simulate fluid behavior. As illustrated in Fig. 15.5, a dashpot is a simple piston–cylinder or a syringe type of arrangement. A force applied on the piston will advance the piston in the direction of the applied force. The speed of the piston is dependent upon the magnitude of the applied force and the friction occurring between the contact surfaces of the piston and cylinder. For a linear dashpot, the applied force and speed (rate of displacement) are linearly proportional, the *coefficient of friction* μ (mu) being the constant of proportionality. If the applied force and the displacement are both in the x direction, then,

$$F = \mu\dot{x}. \tag{15.5}$$

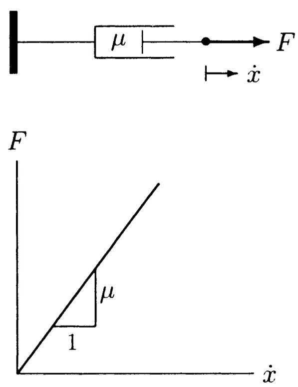

Fig. 15.5 A linear dashpot and its force–displacement rate diagram

In Eq. (15.5), $\dot{x} = \mathrm{d}x/\mathrm{d}t$ is the time rate of change of displacement or the speed.

By comparing Eqs. (15.4) and (15.5), an analogy can be made between linearly viscous fluids and linear dashpots. The stress and the strain rate for a linearly viscous fluid are, respectively, analogous to the force and the displacement rate for a dashpot; and the coefficient of viscosity is analogous to the coefficient of viscous friction for a dashpot. These analogies suggest that dashpots can be used to represent fluid behavior.

15.3 Empirical Models of Viscoelasticity

Springs and dashpots constitute the building blocks of model analyses in viscoelasticity. Springs and dashpots connected to one another in various forms are used to construct empirical viscoelastic models. Springs are used to account for the elastic solid behavior and dashpots are used to describe the viscous fluid behavior (Fig. 15.6). It is assumed that a constantly applied force (stress) produces a constant deformation (strain) in a spring and a constant rate of deformation (strain rate) in a dashpot. The deformation in a spring is completely recoverable upon release of applied forces, whereas the deformation that the dashpot undergoes is permanent.

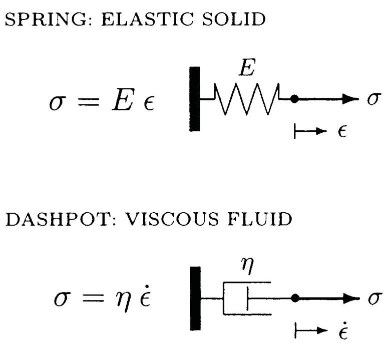

Fig. 15.6 Spring represents elastic and dashpot represents viscous material behaviors

15.3.1 Kelvin–Voight Model

The simplest forms of empirical models are obtained by connecting a spring and a dashpot together in parallel and in series configurations. As illustrated in Fig. 15.7, the *Kelvin–Voight model* is a system consisting of a spring and a dashpot connected in a parallel arrangement. If subscripts "s" and "d" denote the spring and dashpot, respectively, then a stress σ applied to the entire system will produce stresses σ_s and σ_d in the spring and the dashpot. The total stress applied to the system will be shared by the spring and the dashpot such that:

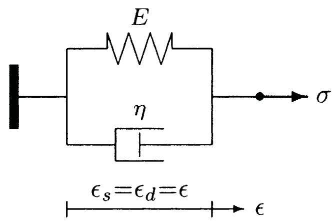

Fig. 15.7 Kelvin–Voight model

$$\sigma = \sigma_s + \sigma_d. \tag{15.6}$$

As the stress σ is applied, the spring and dashpot will deform by an equal amount because of their parallel arrangement. Therefore, the strain ε of the system will be equal to the strains ε_s and ε_d occurring in the spring and the dashpot:

$$\varepsilon = \varepsilon_s = \varepsilon_d. \tag{15.7}$$

The stress–strain relationship for the spring and the stress–strain rate relationship for the dashpot are:

$$\sigma_s = E\varepsilon_s, \tag{15.8}$$

$$\sigma_d = \eta\dot{\varepsilon}_d. \tag{15.9}$$

Substituting Eqs. (15.8) and (15.9) into Eq. (15.6) will yield:

$$\sigma = E\varepsilon_s + \eta\dot{\varepsilon}_d. \tag{15.10}$$

From (15.7), $\varepsilon_s = \varepsilon_d = \varepsilon$. Therefore,

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}. \tag{15.11}$$

Note that the strain rate $\dot{\varepsilon}$ can alternatively be written as $\mathrm{d}\varepsilon/\mathrm{d}t$. Consequently,

$$\sigma = E\varepsilon + \eta\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}. \tag{15.12}$$

Equation (15.12) relates stress to strain and the strain rate for the Kelvin–Voight model, which is a two-parameter (E and η) viscoelastic model. Equation (15.12) is an *ordinary differential equation*. More specifically, it is a first order, linear ordinary differential equation. For a given stress σ, Eq. (15.12) can be solved for the corresponding strain ε. For prescribed strain ε, it can be solved for stress σ.

Note that the review of how to handle ordinary differential equations is beyond the scope of this text. The interested reader is encouraged to review textbooks in "differential equations."

15.3.2 Maxwell Model

As shown in Fig. 15.8, the *Maxwell model* is constructed by connecting a spring and a dashpot in a series. In this case, a stress σ applied to the entire system is applied equally on the spring and the dashpot ($\sigma = \sigma_s = \sigma_d$), and the resulting strain ε is the sum of the strains in the spring and the dashpot ($\varepsilon = \varepsilon_s + \varepsilon_d$). Through stress–strain analyses similar to those carried out for the Kelvin–Voight model, a differential equation relating stresses and strains for the Maxwell model can be derived in the following form:

$$\eta\dot{\sigma} + E\sigma = E\eta\dot{\varepsilon}. \tag{15.13}$$

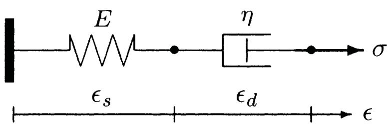

Fig. 15.8 Maxwell model

This is also a first order, linear ordinary differential equation representing a two-parameter (E and η) viscoelastic behavior. For a given stress (or strain), Eq. (15.13) can be solved for the corresponding strain (or stress).

Notice that springs are used to represent the elastic solid behavior, and there is a limit to how much a spring can deform. On the other hand, dashpots are used to represent fluid behavior and are assumed to deform continuously (flow) as long as there is a force to deform them. For example, in the case of a Maxwell model, a force applied will cause both the spring and the dashpot to deform. The deformation of the spring will be finite. The dashpot will keep deforming as long as the force is maintained. Therefore, the overall behavior of the Maxwell model is more like a fluid than a solid, and is known to be a *viscoelastic fluid* model. The deformation of a dashpot connected in parallel to a spring, as in the Kelvin–Voight model, is restricted by the response of the spring to the applied loads. The dashpot in the Kelvin–Voight model cannot undergo continuous deformations. Therefore, the Kelvin–Voight model represents a *viscoelastic solid* behavior.

15.3.3 Standard Solid Model

The Kelvin–Voight solid and Maxwell fluid are the basic viscoelastic models constructed by connecting a spring and a dashpot together. They do not represent any known real material. However, in addition to springs and dashpots, they can be used to construct more complex viscoelastic models, such as the standard solid model. As illustrated in Fig. 15.9, the *standard solid model* is composed of a spring and a Kelvin–Voight solid connected in a series. The standard solid model is a three-parameter (E_1, E_2, and η) model and is used to describe the viscoelastic behavior of a number of biological materials such as the cartilage and the white blood cell membrane. The material function relating the stress, strain, and their rates for this model is:

$$(E_1 + E_2)\sigma + \eta\dot{\sigma} = (E_1E_2\varepsilon + E_1\eta\dot{\varepsilon}). \tag{15.14}$$

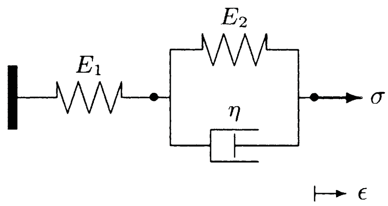

Fig. 15.9 Standard solid model

In Eq. (15.14), $\dot{\sigma} = \mathrm{d}\sigma/\mathrm{d}t$ is the stress rate and $\dot{\varepsilon} = \mathrm{d}\varepsilon/\mathrm{d}t$ is the strain rate. This equation can be derived as follows. As illustrated in Fig. 15.10, the model can be represented by two units, A and B, connected in a series such that unit A is an elastic solid and unit B is a Kelvin–Voight solid. If σ_A and ε_A represent stress and strain in unit A, and σ_B and ε_B are stress and strain in unit B, then,

$$\sigma_A = E_1\varepsilon_A, \tag{i}$$

$$\sigma_B = E_2\varepsilon_B + \eta\frac{\mathrm{d}\varepsilon_B}{\mathrm{d}t} = \left(E_2 + \eta\frac{\mathrm{d}}{\mathrm{d}t}\right)\varepsilon_B. \tag{ii}$$

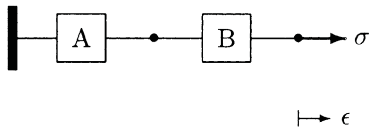

Fig. 15.10 Standard solid model is represented by units A and B

Since units A and B are connected in a series:

$$\varepsilon_A + \varepsilon_B = \varepsilon, \tag{iii}$$

$$\sigma_A = \sigma_B = \sigma. \tag{iv}$$

Substitute Eq. (*iv*) into Eqs. (*i*) and (*ii*) and express them in terms of strains ε_A and ε_B:

$$\varepsilon_A = \frac{\sigma}{E_1}, \tag{v}$$

$$\varepsilon_B = \frac{\sigma}{E_2 + \eta(d/dt)}. \qquad \text{(vi)}$$

Substitute Eqs. (*v*) and (*vi*) into Eq. (*iii*):

$$\frac{\sigma}{E_1} + \frac{\sigma}{E_2 + \eta(d/dT)} = \varepsilon.$$

Employ cross multiplication and rearrange the order of terms to obtain

$$(E_1 + E_2)\sigma + \eta\frac{d\sigma}{dt} = E_1E_2\varepsilon + E_1\eta\frac{d\varepsilon}{dt}$$

15.4 Time-Dependent Material Response

An empirical model for a given viscoelastic material can be established through a series of experiments. There are several experimental techniques designed to analyze the time-dependent aspects of material behavior. As illustrated in Fig. 15.11a, a *creep and recovery* (*recoil*) test is conducted by applying a load (stress σ_o) on the material at time t_0, maintaining the load at a constant level until time t_1, suddenly removing the load at t_1, and observing the material response. As illustrated in Fig. 15.11b, the *stress relaxation* experiment is done by straining the material to a level ε_o and maintaining the constant strain while observing the stress response of the material. In an *oscillatory response* test, a harmonic stress is applied and the strain response of the material is measured (Fig. 15.11c).

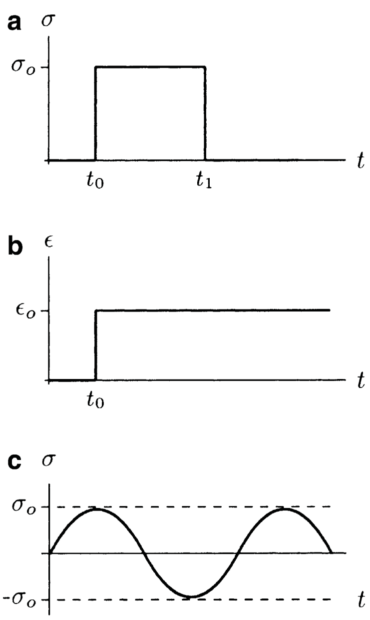

Fig. 15.11 (**a**) Creep and recovery, (**b**) stress relaxation, and (**c**) oscillatory response tests

Consider a viscoelastic material. Assume that the material is subjected to a creep test. The results of the creep test can be represented by plotting the measured strain as a function of time. An empirical viscoelastic model for the material behavior can be established through a series of trials. For this purpose, an empirical model is constructed by connecting a number of springs and dashpots together. A differential equation relating stress, strain, and their rates is derived through the procedure outlined in Sect. 15.3 for the Kelvin–Voight model. The imposed condition in a creep test is $\sigma = \sigma_o$. This condition of constant stress is substituted into the differential equation, which is then solved (integrated) for strain ε. The result obtained is another equation relating strain to stress constant σ_o, the elastic moduli and coefficients of viscosity of the empirical model, and time. For a given σ_o and assigned elastic and viscous moduli, this equation is reduced to a function relating strain to time. This function is then used to plot a strain versus time graph and is compared to the experimentally obtained graph. If the general characteristics of the two (experimental and analytical) curves match, the analyses are furthered to establish the elastic and viscous moduli (material constants) of the material. This is achieved by varying the values of the elastic and viscous moduli in the empirical model until the analytical curve matches the experimental curve as closely as possible. In general, this procedure is called *curve fitting*. If there is no general match between the two curves, the model is abandoned and a new model is constructed and checked.

The result of these mathematical model analyses is an empirical model and a differential equation relating stresses and strains. The stress–strain relationship for the material can be used in conjunction with the fundamental laws of mechanics to analyze the response of the material to different loading conditions.

Note that the deformation processes occurring in viscoelastic materials are quite complex, and it is sometimes necessary to use an array of empirical models to describe the response of a viscoelastic material to different loading conditions. For example, the shear response of a viscoelastic material may be explained with one model and a different model may be needed to explain its response to normal loading. Different models may also be needed to describe the response of a viscoelastic material at low and high strain rates.

15.5 Comparison of Elasticity and Viscoelasticity

There are various criteria with which the elastic and viscoelastic behavior of materials can be compared. Some of these criteria are discussed in this section.

An elastic material has a unique stress–strain relationship that is independent of the time or strain rate. For elastic materials, normal and shear stresses can be expressed as functions of normal and shear strains:

$$\sigma = \sigma(\varepsilon) \quad and \quad \tau = \tau(\gamma).$$

For example, the stress–strain relationships for a linearly elastic solid are $\sigma = E\varepsilon$ and $\tau = G\gamma$, where E and G are constant elastic moduli of the material. As illustrated in Fig. 15.12, a linearly elastic material has a unique normal stress–strain diagram and a unique shear stress–strain diagram.

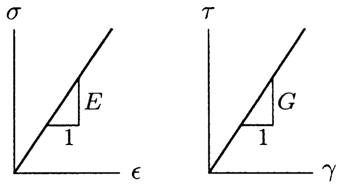

Fig. 15.12 An elastic material has unique normal and shear stress–strain diagrams

Viscoelastic materials exhibit time-dependent material behavior. The response of a viscoelastic material to an applied stress not only depends upon the magnitude of the stress but also on how fast the stress is applied to or removed from the material. Therefore, the stress–strain relationship for a viscoelastic material is not unique but is a function of the time or the rate at which the stresses and strains are developed in the material:

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, \ldots, t) \quad \text{and} \quad \tau = \tau(\gamma, \dot{\gamma}, \ldots, t).$$

Consequently, as illustrated in Fig. 15.13, a viscoelastic material does not have a unique stress–strain diagram.

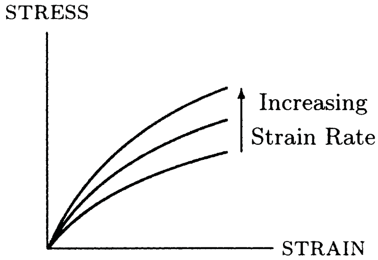

Fig. 15.13 Stress–strain diagram for a viscoelastic material may not be unique

For an elastic body, the energy supplied to deform the body (strain energy) is stored in the body as potential energy. This energy is available to return the body to its original (unstressed) size and shape once the applied stress is removed. As illustrated in Fig. 15.14, the loading and unloading paths for an elastic material coincide. This indicates that there is no loss of energy during loading and unloading.

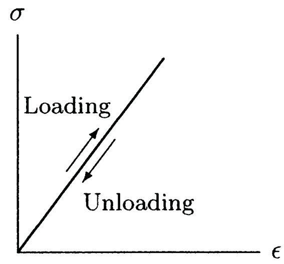

Fig. 15.14 For an elastic material, loading and unloading paths coincide

For a viscoelastic body, some of the strain energy is stored in the body as potential energy and some of it is dissipated as heat. For example, consider the Maxwell model. The energy provided to stretch the spring is stored in the spring while the energy supplied to deform the dashpot is dissipated as heat due to the friction between the moving parts of the dashpot. Once the applied load is removed, the potential energy stored in the spring is available to recover the deformation of the spring, but there is no energy available in the dashpot to regain its original configuration.

Consider the three-parameter standard solid model shown in Fig. 15.9. A typical loading and unloading diagram for this model is shown in Fig. 15.15. The area enclosed by the loading and unloading paths is called the *hysteresis loop*, which represents the energy dissipated as heat during the deformation and recovery phases. This area, and consequently the amount of energy dissipated as heat, is dependent upon the rate of strain employed to deform the body. The presence of the hysteresis loop in the stress–strain diagram for a viscoelastic material indicates that continuous loading and unloading would result in an increase in the temperature of the material.

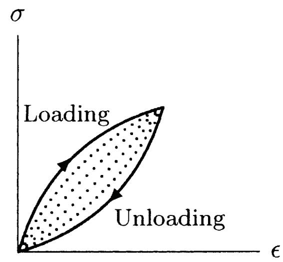

Fig. 15.15 Hysteresis loop

Note here that most of the elastic materials exhibit plastic behavior at stress levels beyond the yield point. For elastic–plastic materials, some of the strain energy is dissipated as heat during plastic deformations. This is indicated with the presence of a hysteresis loop in their loading and unloading diagrams (Fig. 15.16). For such

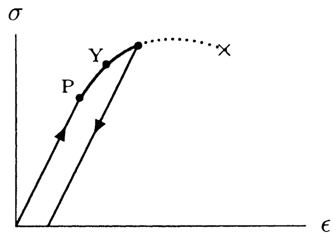

Fig. 15.16 Hysteresis loop for an elastic–plastic material

materials, energy is dissipated as heat only if the plastic region is entered. Viscoelastic materials dissipate energy regardless of whether the strains or stresses are small or large.

Since viscoelastic materials exhibit time-dependent material behavior, the differences between elastic and viscoelastic material responses are most evident under time-dependent loading conditions, such as during the creep and stress relaxation experiments.

As discussed earlier, a creep and recovery test is conducted by observing the response of a material to a constant stress σ_o applied at time t_0 and removed at a later time t_1 (Fig. 15.17a). As illustrated in Fig. 15.17b, such a load will cause a strain $\varepsilon_o = \dfrac{\sigma_o}{E}$ in a linearly elastic material instantly at time t_0. This constant strain will remain in the material until time t_1. At time t_1, the material will instantly and completely recover the deformation. To the same constant loading condition, a viscoelastic material will respond with a strain gradually increasing between times t_0 and t_1. At time t_1, gradual recovery will start. For a viscoelastic solid material, the recovery will eventually be complete (Fig. 15.17c). For a viscoelastic fluid, complete recovery will never be achieved and there will be a residue of deformation left in the material (Fig. 15.17d).

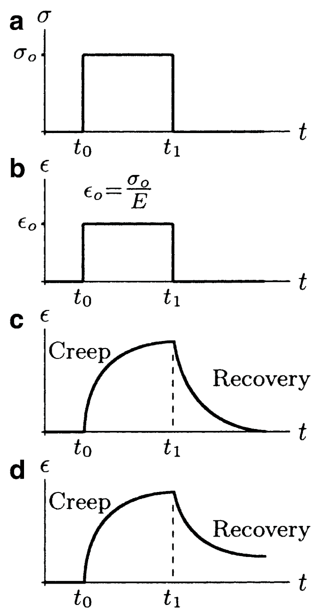

Fig. 15.17 Creep and recovery

As illustrated in Fig. 15.18a, the stress relaxation test is performed by straining a material instantaneously, maintaining the constant strain level ε_o in the material, and observing the response of the material. A linearly elastic material response is illustrated in Fig. 15.18b. The constant stress $\sigma_o = E\varepsilon_o$ developed in the material will remain as long as the strain ε_o is maintained. In other words, an elastic material will not exhibit a stress relaxation behavior. A viscoelastic material, on the other hand, will respond with an initial high stress that will decrease over time. If the material is a viscoelastic solid, the stress level will never reduce to zero (Fig. 15.18c). As illustrated in Fig. 15.18d, the stress will eventually reduce to zero for a viscoelastic fluid.

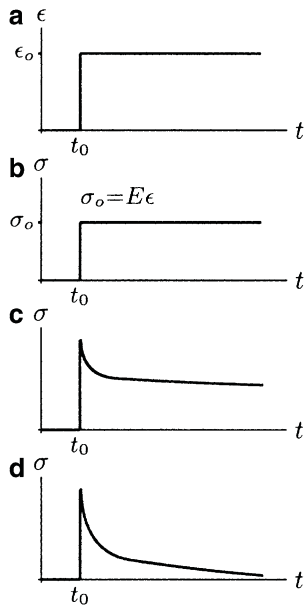

Fig. 15.18 Stress relaxation

Because of their time-dependent material behavior, viscoelastic materials are said to have a "memory." In other words, viscoelastic materials remember the history of deformations they undergo and react accordingly.

Almost all biological materials exhibit viscoelastic properties, and the remainder of this chapter is devoted to the discussion and review of the mechanical properties of biological tissues including bone, tendons, ligaments, muscles, and articular cartilage.

15.6 Common Characteristics of Biological Tissues

One of the objectives of studies in the field of biomechanics is to establish the mechanical properties of biological tissues so as to develop mathematical models that help us describe and further investigate their behavior under various loading conditions. While conducting studies in biomechanics, it has been a common practice to utilize engineering methods and principles, and at the same time to treat biological tissues like engineering materials. However, living tissues have characteristics that are very different than engineering materials. For example, living tissues can be self-adapting and self-repairing. That is, they can adapt to changing mechanical demand by altering their mechanical properties, and they can repair themselves. The mechanical properties of living tissues tend to change with age. Most biological tissues are composite materials (consisting of materials with different properties) with nonhomogeneous and anisotropic properties. In other words, the mechanical properties of living tissues may vary from point to point within the tissue, and their response to forces applied in different directions may be different. For example, values for strength and stiffness of bone may vary between different bones and at different points within the same bone. Furthermore, almost all biological tissues are viscoelastic in nature. Therefore, the strain or loading rate at which a specific test is conducted must also be provided while reporting the results of the strength measurements. These considerations require that most of the mechanical properties reported for living tissues are only approximations and a mathematical model aimed to describe the behavior of a living tissue is usually limited to describing its response under a specific loading configuration.

From a mechanical point of view, all tissues are composite materials. Among the common components of biological tissues, collagen and elastin fibers have the most important mechanical properties affecting the overall mechanical behavior of the tissues in which they appear. *Collagen* is a protein made of crimped fibrils that aggregate into fibers. The mechanical properties of collagen fibrils are such that each fibril can be considered a mechanical spring and each fiber as an assemblage of springs. The primary mechanical function of collagen fibers is to withstand axial tension. Because of their high length-to-diameter ratios (aspect ratio), collagen fibers are not effective under compressive loads. Whenever a fiber is pulled, its crimp straightens, and its length increases. Like a mechanical spring, the energy supplied to stretch the fiber is stored and it is the release of this energy that returns the fiber to its unstretched configuration when the applied load is removed. The individual fibrils of the collagen fibers are surrounded by a gel-like *ground substance* that consists largely of water. Collagen fibers possess a two-phase, solid–fluid, or viscoelastic material behavior with a relatively high tensile strength and poor resistance to compression.

The geometric configuration of collagen fibers and their interaction with the noncollagenous tissue components form the basis of the mechanical properties of biological tissues. Among the noncollagenous tissue components, *elastin* is another fibrous protein with material properties that resemble the properties of rubber. Elastin and microfibrils form elastic fibers that are highly extensible, and their extension is reversible even at high strains. Elastin fibers behave elastically with low stiffness up to about 200% elongation followed by a short region where the stiffness increases sharply until failure (Fig. 15.19). The elastin fibers do not exhibit considerable plastic deformation before failure, and their loading and unloading paths do not show significant hysteresis. In summary, elastin fibers possess a low-modulus elastic material property, while collagen fibers show a higher modulus viscoelastic material behavior.

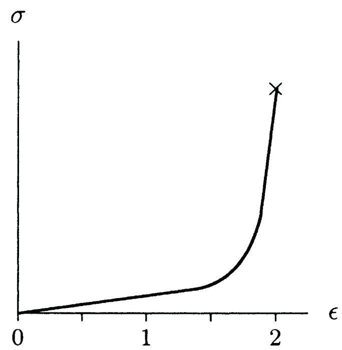

Fig. 15.19 Stress–strain diagram for elastin

15.7 Biomechanics of Bone

Bone is the primary structural element of the human body. Bones form the building blocks of the skeletal system that protects the internal organs, provides kinematic links, provides muscle attachment sites, and facilitates muscle actions and body movements. Bone has unique structural and mechanical properties that allow it to carry out these functions. As compared to other structural materials, bone is also unique in that it is self-repairing. Bone can also alter its shape, mechanical behavior, and mechanical properties to adapt to the changes in mechanical demand. The major factors that influence the mechanical behavior of bone are the composition of bone, the mechanical properties of the tissues comprising the bone, the size and geometry of the bone, and the direction, magnitude, and rate of applied loads.

15.7.1 Composition of Bone

In biological terms, bone is a *connective tissue* that binds together various structural elements of the body. In mechanical terms, bone is a *composite material* with various solid and fluid phases. Bone consists of cells and an organic mineral matrix of fibers and a ground substance surrounding collagen fibers. Bone also contains inorganic substances in the form of mineral salts. The inorganic component of bone makes it hard and relatively rigid, and its organic component provides flexibility and resilience. The composition of bone varies with species, age, sex, type of bone, type of bone tissue, and the presence of bone disease.

At the macroscopic level, all bones consist of two types of tissues (Fig. 15.20). The *cortical* or *compact* bone tissue is a dense material forming the outer shell (cortex) of bones and the diaphysial region of long bones. The *cancellous*, *trabecular*, or *spongy* bone tissue consists of thin plates (trabeculae) in a loose mesh structure that is enclosed by the cortical bone. Bones are surrounded by a dense fibrous membrane called the *periosteum*. The periosteum covers the entire bone except for the joint surfaces that are covered with articular cartilage.

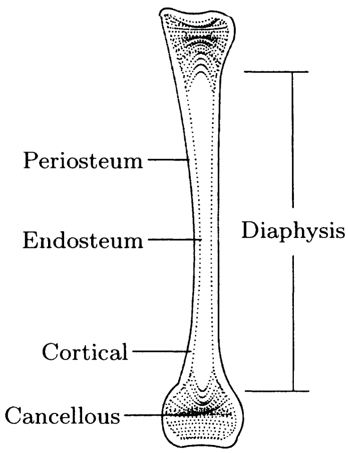

Fig. 15.20 Sectional view of a whole bone showing cortical and cancellous tissues

15.7.2 Mechanical Properties of Bone

Bone is a nonhomogeneous material because it consists of various cells, organic and inorganic substances with different material properties. Bone is an anisotropic material because its mechanical properties are different in different directions. That is, the mechanical response of bone is dependent upon the direction as well as the magnitude of the applied load. For example, the compressive strength of bone is greater than its tensile strength. Bone possesses viscoelastic (time-dependent) material properties. The mechanical response of bone is dependent on the rate at which the loads are applied. Bone can resist rapidly applied loads much better than slowly applied loads. In other words, bone is stiffer and stronger at higher strain rates.

Bone is a complex structural material. The mechanical response of bone can be observed by subjecting it to tension, compression, bending, and torsion. Various tests to implement these conditions were discussed in the previous chapters. These tests can be performed using uniform bone specimens or whole bones. If the purpose is to investigate the mechanical response of a specific bone tissue (cortical or cancellous), then the tests are performed using bone specimens. Testing a whole bone, on the other hand, attempts to determine the "bulk" properties of that bone.

The tensile stress–strain diagram for the cortical bone is shown in Fig. 15.21. This σ–ε curve is drawn using the averages of the elastic modulus, strain hardening modulus, ultimate stress, and ultimate strain values determined for the human femoral cortical bone tested under tensile and compressive loads applied in the longitudinal direction at a moderate strain rate ($\dot{\varepsilon} = 0.05\, s^{-1}$). The σ–ε curve in Fig. 15.21 has three distinct regions. In the initial linearly elastic region, the σ–ε curve is nearly a straight line and the slope of this line is equal to the elastic modulus (E) of the bone, which is about 17 GPa. In the intermediate region, the bone exhibits nonlinear elasto-plastic material behavior. Material yielding also occurs in this region. By the offset method discussed in Chap. 13, the yield strength of the cortical bone for the σ–ε diagram shown in Fig. 15.21 can be determined to be about 110 MPa. In the final region, the bone exhibits a linearly plastic material behavior and the σ–ε diagram is another straight line. The slope of this line is the

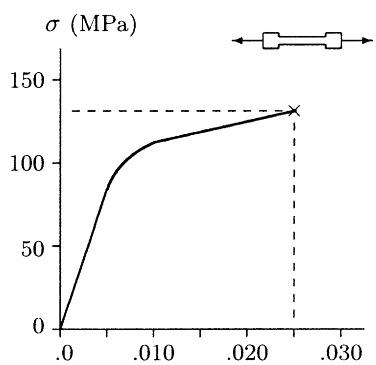

Fig. 15.21 Tensile stress–strain diagram for human cortical bone loaded in the longitudinal direction (strain rate $\dot{\varepsilon} = 0.05\, s^{-1}$)

strain hardening modulus (E') of bone tissue, which is about 0.9 GPa. The bone fractures when the tensile stress is about 128 MPa, for which the tensile strain is about 0.026.

The elastic moduli and strength values for bone are dependent upon many factors including the test conditions such as the rate at which the loads are applied. This viscoelastic nature of bone tissue is demonstrated in Fig. 15.22. The stress–strain diagrams in Fig. 15.22 for different strain rates indicate that a specimen of bone tissue that is subjected to rapid loading (high $\dot{\varepsilon}$) has a greater elastic modulus and ultimate strength than a specimen that is loaded more slowly (low $\dot{\varepsilon}$). Figure 15.22 also demonstrates that the energy absorbed (which is proportional to the area under the σ–ε curve) by the bone tissue increases with an increasing strain rate. Note that during normal daily activities, bone tissues are subjected to a strain rate of about 0.01 s^{-1}.

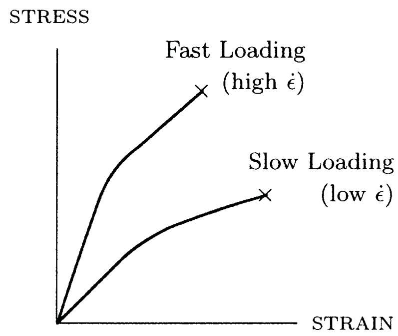

Fig. 15.22 The strain rate-dependent stress–strain curves for cortical bone tissue

The stress–strain behavior of bone is also dependent upon the orientation of bone with respect to the direction of loading. This anisotropic material behavior of bone is demonstrated in Fig. 15.23. Notice that the cortical bone has a larger ultimate strength (stronger) and a larger elastic modulus (stiffer) in the longitudinal direction than the transverse direction. Furthermore, bone specimens loaded in the

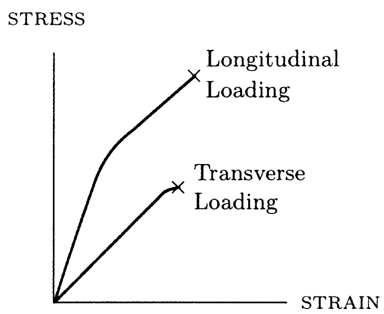

Fig. 15.23 The direction-dependent stress–strain curves for bone tissue

transverse direction fail in a more brittle manner (without showing considerable yielding) as compared to bone specimens loaded in the longitudinal direction. The ultimate strength values for adult femoral cortical bone under various modes of loading, and its elastic and shear moduli are listed in Table 15.1. The ultimate strength values in Table 15.1 demonstrate that the bone strength is highest under compressive loading in the longitudinal direction (the direction of osteon orientation) and lowest under tensile loading in the transverse direction (the direction perpendicular to the longitudinal direction). The elastic modulus of cortical bone in the longitudinal direction is higher than its elastic modulus in the transverse direction. Therefore, cortical bone is stiffer in the longitudinal direction than in the transverse direction.

Table 15.1 Ultimate strength, and elastic and shear moduli for human femoral cortical bone.
(1 GPa = 10^9 Pa, 1 MPa = 10^6 Pa)

LOADING MODE	ULTIMATE STRENGTH
LONGITUDINAL	
Tension	133 MPa
Compression	193 MPa
Shear	68 MPa
TRANSVERSE	
Tension	
Compression	51 MPa
	133 MPa
ELASTIC MODULI, E	
Longitudinal	17.0 GPa
Transverse	11.5 GPa
SHEAR MODULUS, G	3.3 GPa

It should be noted that there is a wide range of variation in values reported for the mechanical properties of bone. It may be useful to remember that the tensile strength of bone is less than 10% of that of stainless steel. Also, the stiffness of bone is about 5% of the stiffness of steel. In other words, for specimens of the same dimension and under the same tensile load, a bone specimen will deform 20 times as much as the steel specimen.

The chemical compositions of cortical and cancellous bone tissues are similar. The distinguishing characteristic of the cancellous bone is its porosity. This physical difference between the two bone tissues is quantified in terms of the *apparent density* of bone, which is defined as the mass of bone tissue present in a unit volume of bone. To a certain degree, both cortical and cancellous bone tissues can be regarded as a single material of variable density. The material properties such as strength and stiffness, and the stress–strain characteristics of cancellous bone depend not only on the apparent density that may be different for different bone types or at different parts of a single bone, but also on the mode of loading. The compressive stress–strain curves (Fig. 15.24) of cancellous bone contain an initial linearly elastic region up to a strain of about 0.05. The material yielding occurs as the trabeculae begin to fracture. This initial elastic region is followed by a plateau region of almost constant stress until fracture, exhibiting a ductile material behavior. By contrast to compact bone, cancellous bone fractures abruptly under tensile forces, showing a brittle material behavior. Cancellous bone is about 25–30% as dense, 5–10% as stiff, and five times as ductile as cortical bone. The energy absorption capacity of cancellous bone is considerably higher under compressive loads than under tensile loads.

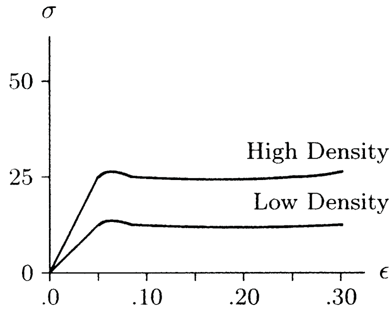

Fig. 15.24 Apparent density-dependent stress–strain curves for cancellous bone tissue

15.7.3 Structural Integrity of Bone

There are several factors that may affect the structural integrity of bones. For example, the size and geometry of a bone determine the distribution of the internal forces throughout the bone, thereby influencing its response to externally applied loads. The larger the bone, the larger the area upon which the internal forces are distributed and the smaller the intensity (stress) of these forces. Consequently, the larger the bone, the more resistant it is to applied loads.

A common characteristic of long bones is their tubular structure in the diaphysial region, which has considerable mechanical advantage over solid circular structures of the same mass. Recall from the previous chapter that the shear stresses in a structure subjected to torsion are inversely proportional with the polar moment of inertia (J) of the cross-sectional area of the structure, and the normal stresses in a structure subjected to bending are inversely proportional to the area moment of inertia (I) of the cross-section of the structure. The larger the polar and area moments of inertia of a structure, the lower the maximum normal stresses due to torsion and bending. Since tubular structures have larger polar and area moments of inertia as compared to solid cylindrical structures of the same volume, tubular structures are more resistant to torsional and bending loads as compared to solid cylindrical structures. Furthermore, a tubular structure can distribute the internal forces more evenly over its cross-section as compared to a solid cylindrical structure of the same cross-sectional area.

Certain skeletal conditions such as osteoporosis can reduce the structural integrity of bone by reducing its apparent density. Small decreases in bone density can generate large reductions in bone strength and stiffness. As compared to a normal bone with the same geometry, an osteoporotic bone will deform easier and fracture at lower loads. The density of bone can also change with aging, after periods of disuse, or after chronic exercise, thereby changing its overall strength. Certain surgical procedures that alter the normal bone geometry may also reduce the strength of bone. Bone defects such as screw holes reduce the load-bearing ability of bone by causing stress concentrations around the defects.

Bone becomes stiffer and less ductile with age. Also with age, the ability of bone to absorb energy and the maximum strain at failure are reduced, and the bone behaves more like dry bone. Although the properties of dry bone may not have any value in orthopedics, it may be important to note that there are differences between bone in its wet and dry states. Dry bone is stiffer, has a higher ultimate strength, and is more brittle than wet bone (Fig. 15.25).

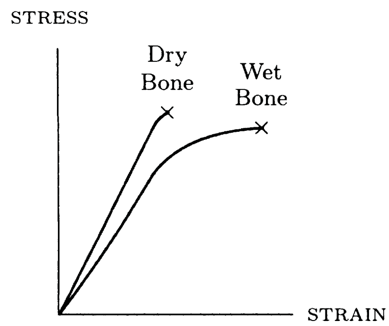

Fig. 15.25 Stress–strain curves for dry and wet bones

15.7.4 Bone Fractures

When bones are subjected to moderate loading conditions, they respond by small deformations that are only present while the loads are applied. When the loads are removed, bones exhibit elastic material behavior by resuming their original (unstressed) shapes and positions. Large deformations occur when the applied loads are high. Bone fractures when the stresses generated in any region of bone are larger than the ultimate strength of bone.

Fractures caused by pure tensile forces are observed in bones with a large proportion of cancellous bone tissue.

Fractures due to compressive loads are commonly encountered in the vertebrae of the elderly, whose bones are weakened as a result of aging. Bone fractures caused by compression occur in the diaphysial regions of long bones. Compressive fractures are identified by their oblique fracture pattern. Long bone fractures are usually caused by torsion and bending. Torsional fractures are identified by their spiral oblique pattern, whereas bending fractures are usually identified by the formation of "butterfly" fragments. Fatigue fracture of bone occurs when the damage caused by repeated mechanical stress outpaces the bone's ability to repair to prevent failure. Bone fractures caused by fatigue are common among professional athletes and dedicated joggers. Clinically, most bone fractures occur as a result of complex, combined loading situations rather than simple loading mechanisms.

15.8 Tendons and Ligaments

Tendons and ligaments are fibrous connective tissues. Tendons help execute joint motion by transmitting mechanical forces (tensions) from muscles to bones. Ligaments join bones and provide stability to the joints. Unlike muscles, which are active tissues and can produce mechanical forces, tendons and ligaments are passive tissues and cannot actively contract to generate forces.

Around many joints of the human body, there is insufficient space to attach more than one or a few muscles. This requires that to accomplish a certain task, one or a few muscles must share the burden of generating and withstanding large loads with intensities (stress) even larger at regions closer to the bone attachments where the cross-sectional areas of the muscles are small. As compared to muscles, tendons are stiffer, have higher tensile strengths, and can endure larger stresses. Therefore, around the joints where the space is limited, muscle attachments to bones are made by tendons. Tendons are capable of supporting very large loads with very small deformations. This property of tendons enables the muscles to transmit forces to bones without wasting energy to stretch tendons.

The mechanical properties of tendons and ligaments depend upon their composition which can vary considerably. The most common means of evaluating the mechanical response of tendons and ligaments is the uniaxial tension test. Figure 15.26 shows a typical tensile stress–strain diagram for tendons. The shape of this curve is the result of the interaction between elastic elastin fibers and the viscoelastic collagen fibers. At low strains (up to about 0.05), less stiff elastic fibers dominate and the crimp of the collagen fibers straightens, requiring very little force to stretch the tendon. The tendon becomes stiffer when the crimp is straightened. At the same time, the fluid-like ground substance in the collagen fibers tends to flow. At higher strains, therefore, the stiff and viscoelastic nature of the collagen fibers begins to take an increasing portion of the applied load. Tendons are believed to function in the body at strains of up to about 0.04, which is believed to be their yield strain (ε_y). Tendons rupture at strains of about 0.1 (ultimate strain, ε_u), or stresses of about 60 MPa (ultimate stress, σ_u).

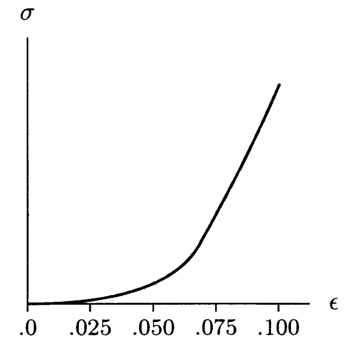

Fig. 15.26 Tensile stress–strain diagram for tendon

Note that the shape of the stress–strain curve in Fig. 15.26 is such that the area under the curve is considerably small. In other words, the energy stored in the tendon to stretch the tendon to a stress level is much smaller than the energy stored to stretch a linearly elastic material (with a stress–strain diagram that is a straight line) to the same stress level. Therefore, the tendon has higher resilience than linearly elastic materials.

The time-dependent, viscoelastic nature of the tendon is illustrated in Figs. 15.27 and 15.28. When the tendon is stretched rapidly, there is less chance for the ground substance to flow, and consequently, the tendon becomes stiffer. The hysteresis loop shown in Fig. 15.28 demonstrates the time-dependent loading and unloading behavior of the tendon. Note that more work is done in stretching the tendon than is recovered when the tendon is allowed to relax, and therefore, some of the energy is dissipated in the process.

The mechanical role of ligaments is to transmit forces from one bone to another. Ligaments also have a stabilizing

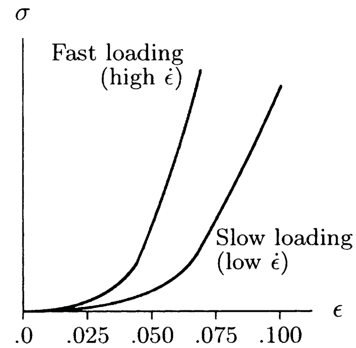

Fig. 15.27 The strain rate-dependent stress–strain curves for tendon

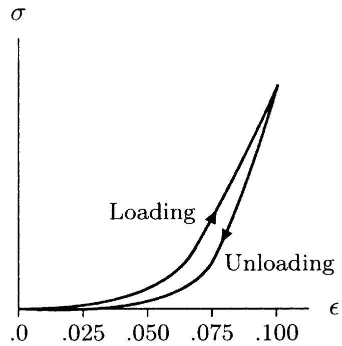

Fig. 15.28 The hysteresis loop of stretching and relaxing modes of the tendon

role for the skeletal joints. The composition and structure of ligaments depend upon their function and position within the body. Like tendons they are composite materials containing crimped collagen fibers surrounded by ground substance. As compared to tendons, they often contain a greater proportion of elastic fibers that accounts for their higher extensibility but lower strength and stiffness. The mechanical properties of ligaments are qualitatively similar to those of tendons. Like tendons, they are viscoelastic and exhibit hysteresis, but deform elastically up to strains of about $\varepsilon_y = 0.25$ (about five times as much as the yield strain of tendons) and stresses of about $\sigma_y = 5\,\text{MPa}$. They rupture at a stress of about 20 MPa.

Since tendons and ligaments are viscoelastic, some of the energy supplied to stretch them is dissipated by causing the flow of the fluid within the ground substance, and the rest of the energy is stored in the stretched tissue. Tendons and ligaments are tough materials and do not rupture easily. Most common damages to tendons and ligaments occur at their junctions with bones.

15.9 Skeletal Muscles

There are three types of muscles: skeletal, smooth, and cardiac. Smooth muscles line the internal organs, and cardiac muscles form the heart. Here, we are concerned with the characteristics of the skeletal muscles, each of which is attached, via aponeuroses and/or tendons, to at least two bones causing and/or controlling the relative movement of one bone with respect to the other. When its fibers contract under the stimulation of a nerve, the muscle exerts a pulling effect on the bones to which it is attached. *Contraction* is a unique ability of the muscle tissue, which is defined as the development of tension in the muscle. Muscle contraction can occur as a result of muscle shortening (concentric contraction) or muscle lengthening (eccentric contraction), or it can occur without any change in the muscle length (static or isometric contraction).

The skeletal muscle is composed of muscle fibers and myofibrils. Myofibrils in turn are made of contractile elements: *actin* and *myosin* proteins. Actin and myosin appear in bands or filaments. Several relatively thick myosin filaments interact across cross-bridges with relatively thin actin filaments to form the basic structure of the contractile element of the muscle, called the *sarcomere* (Fig. 15.29). Many sarcomere elements connected in a series arrangement form the contractile element (motor unit) of the muscle. It is within the sarcomere that the muscle force (tension) is generated, and where muscle shortening and lengthening takes place. The active contractile elements of the muscle are contained within a fibrous passive connective tissue, called *fascia*. Fascia encloses the muscles, separates them into layers, and connects them to tendons.

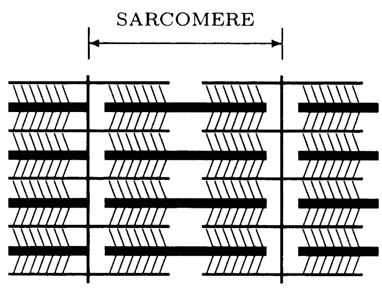

Fig. 15.29 Basic structure of the contractile element of muscle (*thick lines* represent myosin filaments, *thin horizontal lines* are actin filaments, and *cross-hatched lines* are cross-bridges)

The force and torque developed by a muscle is dependent on many factors, including the number of motor units within the muscle, the number of motor units recruited, the manner in which the muscle changes its length, the velocity of muscle contraction, and the length of the lever arm of the muscle force. For muscles, two different forces can be distinguished. *Active tension* is the force produced by the contractile elements of the muscle and is a result of voluntary muscle contraction. *Passive tension*, on the other hand, is the force developed within the connective muscle tissue when the muscle length surpasses its resting length. The net tensile force in a muscle is dependent on the force–length characteristics of both the active and passive components of the muscle. A typical tension versus muscle length diagram is shown in Fig. 15.30. The number of cross-bridges between the filaments is maximum, and therefore, the active tension (T_a) is maximum at the resting length (l_o) of the muscle. As the muscle lengthens, the filaments are pulled apart, the number of cross-bridges is reduced and the active tension is decreased. At full length, there are no cross-bridges and the active tension reduces to zero. As the muscle shortens, the cross-bridges overlap and the active tension is again reduced. When the muscle is at its resting length or

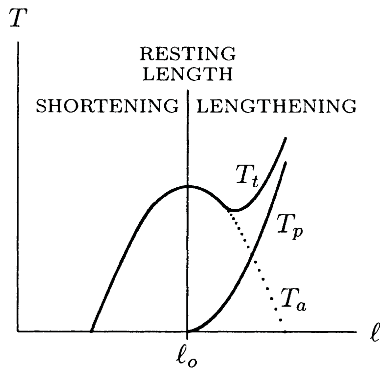

Fig. 15.30 Muscle force (T) versus muscle length (ℓ)

less, the passive (connective) component of the muscle is in a loose state with no tension. As the muscle lengthens, a passive tensile force (T_p) builds up in the connective tissues. The force–length characteristic of this passive component resembles that of a nonlinear spring. Passive tensile force increases at an increasing rate as the length of the muscle increases. The overall, total, or net muscle force (T_t) that is transmitted via tendons is the sum of the forces in the active and passive elements of the muscle. Note here that for a given muscle, the tension–length diagram is not unique but dependent on the number of motor units recruited. The magnitude of the active component of the muscle force can vary depending on how the muscle is excited, and usually expressed as the percentage of the maximum voluntary contraction.

The force generated by a contracting muscle is usually transmitted to a bone through a tendon. There is a functional reason for tendons to make the transfer of forces from muscles to bones. As compared to tendons, muscles have lower tensile strengths. The relatively low ultimate strength requires muscles to have relatively large cross-sectional areas in order to transmit sufficiently high forces without tearing. Tendons are better designed to perform this function.

15.10 Articular Cartilage

Cartilage covers the articulating surfaces of bones at the diarthrodial (synovial) joints. The primary function of cartilage is to facilitate the relative movement of articulating bones. Cartilage reduces stresses applied to bones by increasing the area of contact between the articulating surfaces and reduces bone wear by reducing the effects of friction.

Cartilage is a two-phase material consisting of about 75% water and 25% organic solid. A large portion of the solid phase of the cartilage material is made up of collagen fibers. The remaining ground substance is mainly proteoglycan (hydrophilic molecules). Collagen fibers are relatively strong and stiff in tension, while proteoglycans are strong in compression. The solid–fluid composition of cartilage makes it a viscoelastic material.

The mechanical properties of cartilage under various loading conditions have been investigated using a number of different techniques. For example, the response of the human patella to compressive loads has been investigated by using an *indentation test* in which a small cylindrical or hemispherical indenter is pressed into the articulating surface, and the resulting deformation is recorded (Fig. 15.31a). A typical result of an indentation test is shown in Fig. 15.31b. When a constant magnitude load is applied, the material initially responds with a relatively large elastic deformation. The applied load causes pressure gradients to occur in the interstitial fluid, and the variations in pressure cause the fluid to flow through and out of the cartilage matrix. As the load is maintained, the amount of deformation increases at a decreasing rate. The deformation tends toward an equilibrium state as the pressure variations within the fluid are dissipated. When the applied load is removed (unloading phase), there is an instantaneous elastic recovery (recoil) that is followed by a more gradual recovery leading to complete recovery. This creep–recovery response of cartilage may be qualitatively represented by the three-parameter viscoelastic solid model (Fig. 15.32), which consists of a linear spring and a Kelvin–Voight unit connected in series.

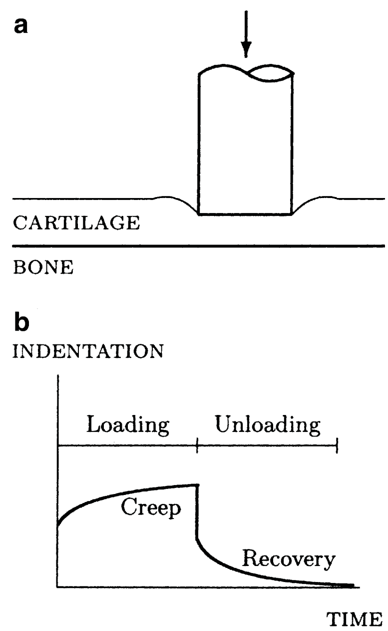

Fig. 15.31 Indentation test

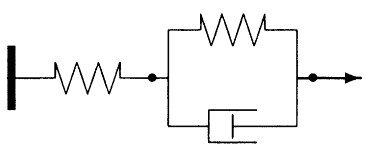

Fig. 15.32 The standard solid model has been used to represent the creep–recovery behavior of cartilage

Another experiment designed to investigate the response of cartilage to compressive loading conditions is the *confined compression test* illustrated in Fig. 15.33. In this test, the specimen is confined in a rigid cylindrical die and loaded with a rigid permeable block. The compressive load causes pressure variations in the interstitial fluid and consequent fluid exudation. Eventually, the pressure variations dissipate and equilibrium is reached. The state at which the equilibrium is reached is indicative of the compressive stiffness of the cartilage. The compressive stiffness and resistance of cartilage depend upon the water and proteoglycan content of the tissue. The higher the proteoglycan content, the higher the compressive resistance of the tissue.

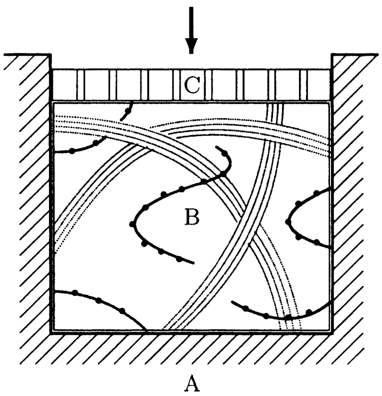

Fig. 15.33 Confined compression test. A is the rigid die, B is the specimen, and C is the permeable block

During daily activities, the articular cartilage is subjected to tensile and shear stresses as well as compressive stresses. Under tension, cartilage responds by realigning the collagen fibers that carry the tensile loads applied to the tissue. The tensile stiffness and strength of cartilage depend on the collagen content of the tissue. The higher the collagen content, the higher the tensile strength of cartilage. Shear stresses on the articular cartilage are due to the frictional forces between the relative movement of articulating surfaces. However, the coefficient of friction for synovial joints is so low (of the order 0.001–0.06) that friction has an insignificant effect on the stress resultants acting on the cartilage.

Both structural (such as intraarticular fracture) and anatomical abnormalities (such as rheumatoid arthritis and acetabular dysplasia) can cause cartilage damage, degeneration, wear, and failure. These abnormalities can change the load-bearing ability of the joint by altering its mechanical properties. The importance of the load-bearing ability of the cartilage and maintaining its mechanical integrity may become clear if we consider that the magnitude of the forces involved at the human hip joint is about five times body weight during ordinary walking (much higher during running or jumping). The hip contact area over which these forces are applied is about 15 cm^2 (0.0015 m^2). Therefore, the compressive stresses (pressures) involved are of the order 3 MPa for an 85 kg person.

15.11 Discussion

Here we have covered, very briefly, the mechanical properties of selected biological tissues. We believe that the knowledge of the mechanical properties and structural behavior of biological tissues is an essential prerequisite for any experimental or theoretical analysis of their physiological function in the body. We are aware of the fact that the proper coverage of each of these topics deserves at least a full chapter. Our purpose here was to provide a summary, to illustrate how biological phenomena can be described in terms of the mechanical concepts introduced earlier, and hope that the interested reader would refer to more complete sources of information to improve his or her knowledge of the subject matter.

15.12 Exercise Problem

Problem 15.1 Complete the following definitions with appropriate expressions.

(a) Elastic materials show time-independent material behavior. Elastic materials deform ________ when they are subjected to externally applied loads.

(b) Time-dependent material behavior is known as __________.

(c) Elasticity is a solid material behavior, whereas ____________ is a fluid property and is a measure of resistance to flow.

(d) For a viscoelatic material, stress is not only a function of strain, but also a function of _________.

(e) _________ and _________ are basic mechanical elements that are used to simulate elastic solid and viscous fluid behaviors, respectively.

(f) The _________ is a viscoelastic model consisting of a spring and a dashpot connected in a parallel arrangement.

(g) The _________ is a viscoelastic model consisting of a spring and a dashpot connected in a series arrangement.

(h) The _________ is a viscoelastic model consisting of a spring and a Kelvin–Voight solid connected in a series.

(i) A _________ test is conducted by applying a load on the material, maintaining the load at a constant level for some time, suddenly removing the load, and observing the material response.