BME 311
Biomechanics
Week 1
Moment and Torque

Textbook:
Basic Biomechanics of the Musculoskeletal System, 4th Ed., 2012, Edited by Margareta Nordin, Victor H. Frankel

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Example 3.1 Figure 3.13 illustrates a person preparing to dive into a pool. The horizontal diving board has a uniform thickness, mounted to the ground at point O, has a mass of 120 kg, and $l = 4$ m in length. The person has a mass of 90 kg and stands at point B which is the free-end of the board. Point A indicates the location of the center of gravity of the board. Point A is equidistant from points O and B.

Determine the moments generated about point O by the weights of the person and the board. Calculate the net moment about point O

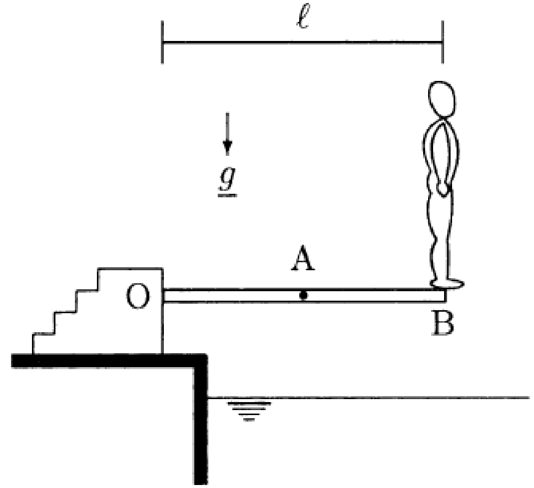

Fig. 3.13 A person is preparing to dive

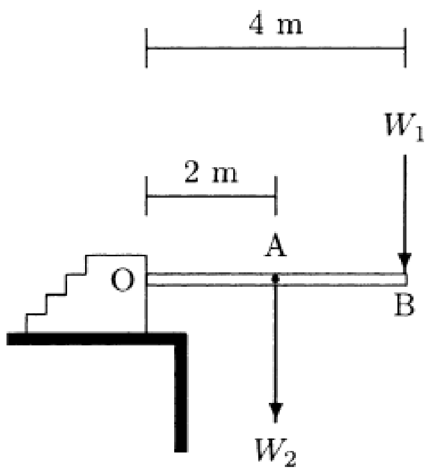

Fig. 3.14 Forces acting on the diving board

$$W_1 = m_1 g = (90\text{ kg})(9.8\text{ m/s}^2) = 882\text{ N}$$
$$W_2 = m_2 g = (120\text{ kg})(9.8\text{ m/s}^2) = 1,176\text{ N}.$$

The person is standing at point B. Therefore, the weight $\underline{W}_1$ of the person is applied on the board at point B. The mass of the diving board produces a force system distributed over the entire length of the board. The resultant of this distributed force system is equal to the weight $\underline{W}_2$ of the board. For practical purposes and since the board has a uniform thickness, we can assume that the weight of the board is a concentrated force acting at A which is the center of gravity of the board.

As shown in Fig. 3.14, weights $\underline{W}_1$ and $\underline{W}_2$ act vertically downward or in the direction of gravitational acceleration. The diving board is horizontal. Therefore, the distance between points O and B(l) is the length of the moment arm for $\underline{W}_1$ and the distance between points O and A($l/2$) is the length of the moment arm for $\underline{W}_2$. Therefore, moments $\underline{M}_1$ and $\underline{M}_2$ due to $\underline{W}_1$ and $\underline{W}_2$ about point O are

$$M_1 = lW_1 = (4\text{ m})(882\text{ N}) = 3,528\text{ N-m} \quad (\text{cw})$$
$$M_2 = \frac{l}{2}W_2 = (2\text{ m})(1,176\text{ N}) = 2,352\text{ N-m} \quad (\text{cw}).$$

Since both moments have a clockwise direction, the net moment must have a clockwise direction as well. The magnitude of the net moment about point O is

$$M_{\text{net}} = M_1 + M_2 = 880\text{ N-m} \quad (\text{cw}).$$

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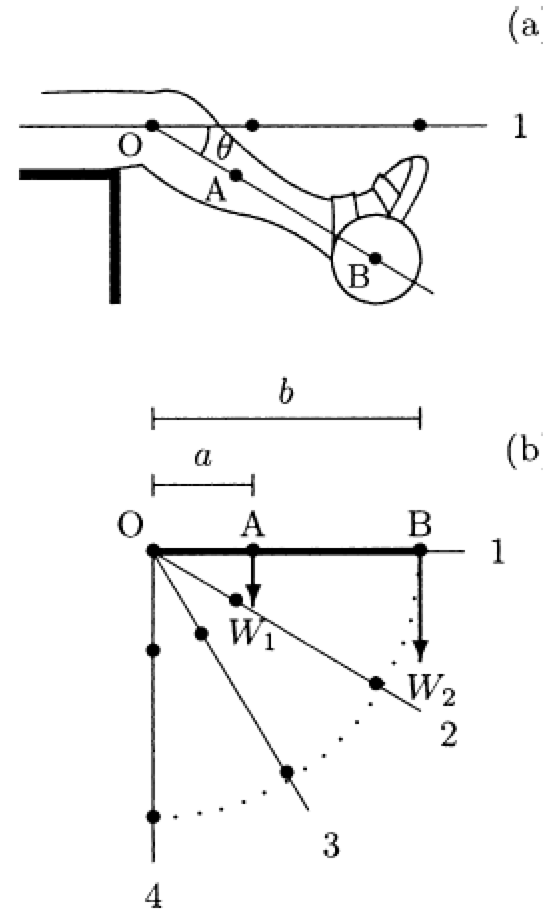
Example 3.2 As illustrated in Fig. 3.15a, consider an athlete wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen quadriceps muscles. The weight of the athlete's lower leg is $W_1 = 50$ N and the weight of the boot is $W_2 = 100$ N. As measured from the knee joint at point O, the center of gravity (point A) of the lower leg is located at a distance $a = 20$ cm and the center of gravity (point B) of the weight boot is located at a distance $b = 50$ cm.

Determine the net moment generated about the knee joint when the lower leg is extended horizontally (position 1), and when the lower leg makes an angle of 30° (position 2), 60° (position 3), and 90° (position 4) with the horizontal (Fig. 3.15b).



$$
\begin{aligned}
M_O &= aW_1 + bW_2 \\
&= (0.20)(50) + (0.50)(100) \\
&= 60 \text{ N-m} \quad (\text{cw}).
\end{aligned}
$$

Figure 3.16 illustrates the external forces acting on the lower leg and their moment arms (d_1 and d_2) when the lower

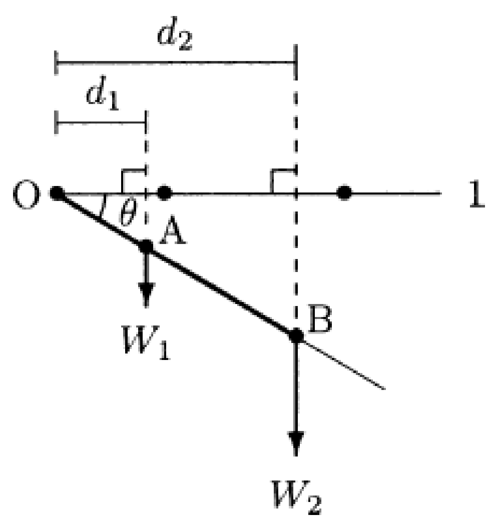
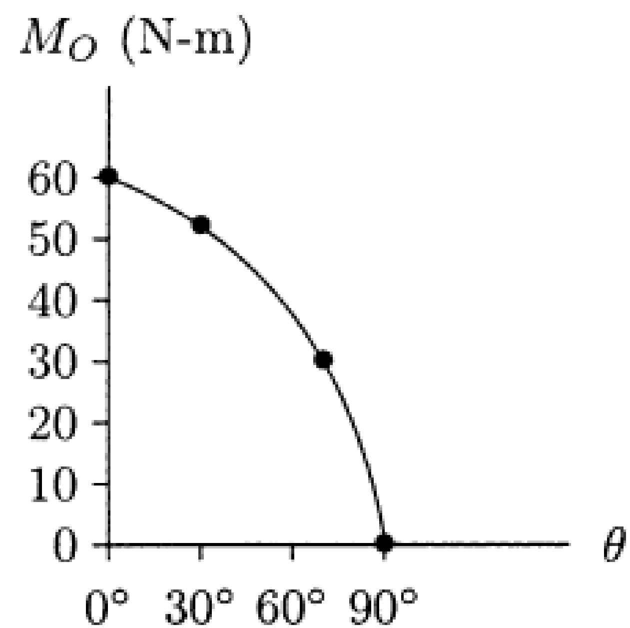


Fig. 3.16 Forces and moment arms when the lower leg makes an angle θ with the horizontal



Quiz

Example 3.3 Figure 3.18a illustrates an athlete doing shoulder muscle strengthening exercises by lowering and raising a barbell with straight arms. The position of the arms when they make an angle θ with the vertical is simplified in Fig. 3.18b. Point O represents the shoulder joint, A is the center of gravity of one arm, and B is a point of intersection of the centerline of the barbell and the extension of line OA. The distance between points O and A is $a = 24$ cm and the distance between points O and B is $b = 60$ cm. Each arm weighs $W_1 = 50$ N and the total weight of the barbell is $W_2 = 300$ N.

Determine the net moment due to $\underline{W}_1$ and $\underline{W}_2$ about the shoulder joint as a function of θ, which is the angle the arm makes with the vertical. Calculate the moments for $\theta = 0°$, $15°$, $30°$, $45°$, and $60°$.

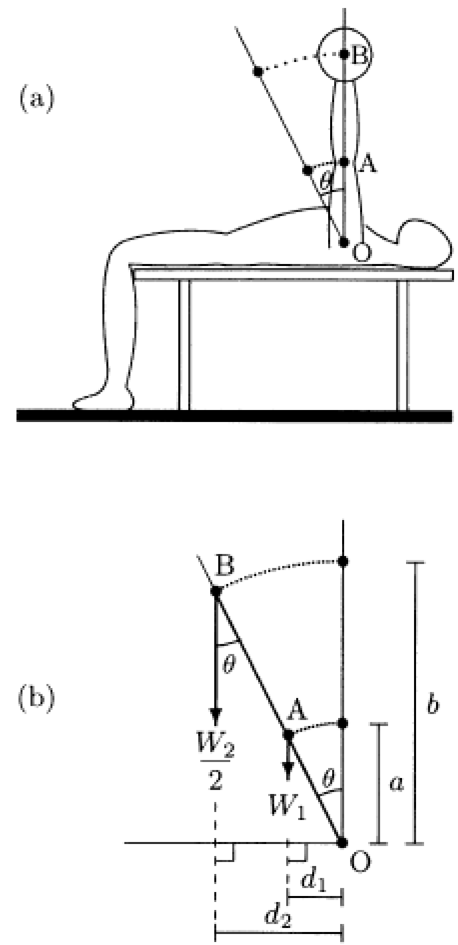


Fig. 3.18 An exercise to strengthen the shoulder muscles, and a simple model of the arm

Quiz: Solution

To calculate the moments generated about the shoulder joint by $\underline{W}_1$ and $\underline{W}_2$, we need to determine the moment arms d_1 and d_2 of forces $\underline{W}_1$ and $\underline{W}_2$ relative to point O. From the geometry of the problem (Fig. 3.18b), the lengths of the moment arms are

$$d_1 = a \sin \theta$$
$$d_2 = b \sin \theta.$$

Since the athlete is using both arms, the total weight of the barbell is assumed to be shared equally by each arm. Also note that relative to the shoulder joint, both the weight of the arm and the weight of the barbell are trying to rotate the arm in the counterclockwise direction. Moments $\underline{M}_1$ and $\underline{M}_2$ due to $\underline{W}_1$ and $\underline{W}_2$ about point O are

$$M_1 = d_1 W_1 = a W_1 \sin \theta = (0.24)(50) \sin \theta = 12 \sin \theta$$
$$M_2 = d_2 \frac{W_2}{2} = b \frac{W_2}{2} \sin \theta = (0.60) \left(\frac{300}{2} \right) \sin \theta = 90 \sin \theta.$$

Since both moments are counterclockwise, the net moment must be counterclockwise as well. Therefore, the net moment $\underline{M}_O$ generated about the shoulder joint is

$$M_O = M_1 + M_2 = 12 \sin \theta + 90 \sin \theta = 102 \sin \theta \text{ N-m} \quad \text{(ccw)}.$$

To determine the magnitude of the moment about point O, for $\theta = 0°$, $15°$, $30°$, $45°$, and $60°$, all we need to do is evaluate the sines and carry out the multiplications. The results are provided in Table 3.2.

Table 3.2 Moment about the shoulder joint (Example 3.3)

θ (°)	$\sin \theta$	M_O (N-m)
0	0.000	0.0
15	0.259	26.4
30	0.500	51.0
45	0.707	72.1
60	0.866	88.3